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Pionic parton distributions revisited

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Abstract. Using constituent quark model constraints we calculate the gluon and sea–quark content of pions solely in terms of their valence density (fixed by πN Drell–Yan data) and the known sea and gluon distributions of the nucleon, using the most recent updated valence–like input parton densities of the nucleon. The resulting small–x dynamical QCD predictions for $g^\pi(x,Q^2)$ and $\bar{q}^{\,\pi}(x,Q^2)$ are unique and parameter free. Simple analytic parametrizations of the resulting parton distributions of the pion are presented in LO and NLO. These results and parametrizations will be important, among other things, for updated formulations of the parton distributions of real and virtual photons.

The parton content of the pion is poorly known at present. The main experimental source about these distributions is mainly due to data of Drell–Yan dilepton production in π^-–tungsten reactions [1–3], which determine the shape of the pionic valence density $v^\pi(x,Q^2)$ rather well, and due to measurements of direct photon production in $\pi^\pm p \to \gamma X$ [1,4] which constrain the pionic gluon distribution $g^\pi(x,Q^2)$ only in the large–x region [5]. In general, however, present data are not sufficient for fixing g^π uniquely, in particular the pionic sea density $\bar{q}^{\,\pi}(x,Q^2)$ remains entirely unconstrained experimentally. Therefore we have previously [6] utilized a constituent quark model [7] to relate $\bar{q}^{\,\pi}$ and g^π to the much better known radiatively generated parton distributions $f^p(x,Q^2)$ of the proton [8]. These relations arise as follows: describing the constituent quark structure of the proton $p = UUD$ and the pion, say $\pi^+ = U\bar{D}$, by the scale (Q^2) independent distributions $U^{p,\pi^+}(x)$, $D^p(x)$ and $\bar{D}^{\,\pi^+}(x)$, and their universal (i.e. hadron independent) partonic content by $v_c(x,Q^2)$, $g_c(x,Q^2)$ and $\bar{q}_c(x,Q^2)$, the usual parton content of the proton and the pion is then given by

$$f^p(x,Q^2) = \int_x^1 \frac{dy}{y}\,[U^p(y)+D^p(y)]\,f_c\left(\frac{x}{y},Q^2\right) \qquad (1)$$

$$f^\pi(x,Q^2) = \int_x^1 \frac{dy}{y}\left[U^{\pi^+}(y)+\bar{D}^{\,\pi^+}(y)\right] f_c\left(\frac{x}{y},Q^2\right) \qquad (2)$$

where $f = v,\,\bar{q},\,g$ with $v^p = u_v^p + d_v^p$, $\bar{q}^{\,p} = (\bar{u}^{\,p}+\bar{d}^{\,p})/2$, $v^\pi = u_v^{\pi^+} + \bar{d}_v^{\,\pi^+}$, $\bar{q}^{\,\pi} = (\bar{u}^{\,\pi^+} + d^{\pi^+})/2$ and $\bar{u}^{\,\pi^+} = d^{\pi^+}$ due to ignoring minor SU(2)$_{\rm flavor}$ breaking effects in the pion 'sea' distributions. Assuming these relations to apply at the low resolution scale $Q^2 = \mu^2$ ($\mu^2_{\rm LO} = 0.23$ GeV^2, $\mu^2_{\rm NLO} = 0.34$ GeV^2) of [8] where the strange quark content was considered to be negligible,

$$s^p(x,\mu^2) = \bar{s}^{\,p}(x,\mu^2) = s^\pi(x,\mu^2) = \bar{s}^{\,\pi}(x,\mu^2) = 0, \quad (3)$$

one obtains from (1) and (2) the constituent quark independent relations [6]

$$\frac{v^\pi(n,\mu^2)}{v^p(n,\mu^2)} = \frac{\bar{q}^{\,\pi}(n,\mu^2)}{\bar{q}^{\,p}(n,\mu^2)} = \frac{g^\pi(n,\mu^2)}{g^p(n,\mu^2)} \qquad (4)$$

where for convenience we have taken the Mellin n-moments of (1) and (2), i.e. $f(n,Q^2) \equiv \int_0^1 x^{n-1} f(x,Q^2)dx$. Thus, as soon as $v^\pi(x,\mu^2)$ is reasonably well determined from experiment, our basic relations (4) uniquely fix the gluon and sea densities of the pion in terms of the rather well known parton distributions of the proton:

$$g^\pi(n,\mu^2) = \frac{v^\pi(n,\mu^2)}{v^p(n,\mu^2)}\,g^p(n,\mu^2),$$
$$\bar{q}^{\,\pi}(n,\mu^2) = \frac{v^\pi(n,\mu^2)}{v^p(n,\mu^2)}\,\bar{q}^{\,p}(n,\mu^2). \qquad (5)$$

Furthermore, the sum rules [6]

$$\int_0^1 v^\pi(x,Q^2)dx = 2 \qquad (6)$$

$$\int_0^1 x v^\pi(x,Q^2)dx = \int_0^1 x v^p(x,Q^2)dx \qquad (7)$$

impose strong constraints on $v^\pi(x,\mu^2)$ which are very useful for its almost unambiguous determination from the πN Drell–Yan data. Notice that (7), together with (4), implies the energy–momentum sum rule for f^π to be manifestly satisfied. In addition, (7) implies that the valence quarks in the proton and the pion carry similar total fractional momentum as suggested by independent analyses within the framework of the radiative parton model [5,8].

The relations in (5) imply that any updating of $f^p(x,\mu^2)$ yields a corresponding updating of $f^\pi(x,\mu^2)$. Recently an updating of $f^p(x,\mu^2)$ within the framework of the radiative (dynamical) parton model was undertaken [9] utilizing additional improved data on $F_2^p(x,Q^2)$ from HERA [10,11] and a somewhat increased $\alpha_s(M_Z^2) = 0.114$ resulting in a slight increase in μ^2 ($\mu^2_{\rm LO} = 0.26$ GeV^2, $\mu^2_{\rm NLO} = 0.40$ GeV^2). An improved treatment of the running $\alpha_s(Q^2)$ at low Q^2 was furthermore implemented by solving in NLO($\overline{\rm MS}$)

$$\frac{d\alpha_s(Q^2)}{d\,ln\,Q^2} = -\frac{\beta_0}{4\pi}\,\alpha_s^2(Q^2) - \frac{\beta_1}{16\pi^2}\,\alpha_s^3(Q^2) \tag{8}$$

numerically [9] rather than using the approximate NLO solution

$$\frac{\alpha_s(Q^2)}{4\pi} \simeq \frac{1}{\beta_0\,ln\,(Q^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\,\frac{ln\,ln\,(Q^2/\Lambda^2)}{ln^2\,(Q^2/\Lambda^2)} \tag{9}$$

as done in [5,6,8], which is sufficiently accurate only for $Q^2 \gtrsim m_c^2 \simeq 2$ GeV^2 [9]. The LO and NLO evolutions of $f^\pi(n,Q^2)$ to $Q^2 > \mu^2$ are performed in Mellin n–moment space, followed by a straightforward numerical Mellin–inversion [12] to Bjorken-x space. It should be noted that the evolutions are always performed in the fixed (light) $f = 3$ flavor factorization scheme [13,6,8,9], i.e. we refrain from generating radiatively massless 'heavy' quark densities $h^\pi(x,Q^2)$ where $h = c$, b, etc., in contrast to [5]. Hence heavy quark contributions have to be calculated in fixed–order perturbation theory via, e.g., $g^\pi g^p \to h\bar{h}$, $\bar{u}^{\,\pi} u^p \to h\bar{h}$, etc. (Nevertheless, rough estimates of 'heavy' quark effects, valid to within a factor of 2, say, can be easier obtained with the help of the massless densities $c^\pi(x,Q^2)$ and $b^\pi(x,Q^2)$ given in [5].)

Using all these modified ingredients together with the new updated [9] $f^p(x,\mu^2)$ in our basic predictions in (5), the present reanalysis of the available Drell–Yan data [2], closely following the procedure described in [6], yields

$$v^\pi_{\rm LO}(x,\mu^2_{\rm LO}) = 1.129x^{-0.496}(1-x)^{0.349} \times(1+0.153\sqrt{x}) \tag{10}$$

$$v^\pi_{\rm NLO}(x,\mu^2_{\rm NLO}) = 1.391x^{-0.447}(1-x)^{0.426} \tag{11}$$

where [9] $\mu^2_{\rm LO} = 0.26$ GeV^2 and $\mu^2_{\rm NLO} = 0.40$ GeV^2. These updated input valence densities correspond to total momentum fractions

$$\int_0^1 x\,v^\pi_{\rm LO}(x,\mu^2_{\rm LO})dx = 0.563 \tag{12}$$

$$\int_0^1 x\,v^\pi_{\rm NLO}(x,\mu^2_{\rm NLO})dx = 0.559 \tag{13}$$

as dictated by the valence densities of the proton [9] via (7). Our new updated input distributions in (10), (11) and (5) are rather different than the original GRV$_\pi$ input [5] in Fig. 1 which is mainly due to the vanishing sea input of GRV$_\pi$ in contrast to the present one in (5). On the other hand, our updated input in Fig. 1 is, as expected, rather similar to the one of [6]. In both cases, however, the valence and gluon distributions become practically indistinguishable from our present updated ones at scales relevant for present Drell–Yan dimuon and direct–γ production data, $Q^2 \equiv M^2_{\mu^+\mu^-} \simeq 20$ GeV^2, as illustrated in Fig. 2. Therefore our present updated pionic distributions give an equally good description of all available πN Drell–Yan data as the ones shown in [6]. Notice that the different gluon distributions presented in Fig. 2 can not be discriminated by present direct–photon production data [4] due to the uncertainty of the theoretically calculated cross section arising from variations of the chosen factorization scale and from possible intrinsic k_T contributions, cf. for example L. Apanasevich et al. [4].

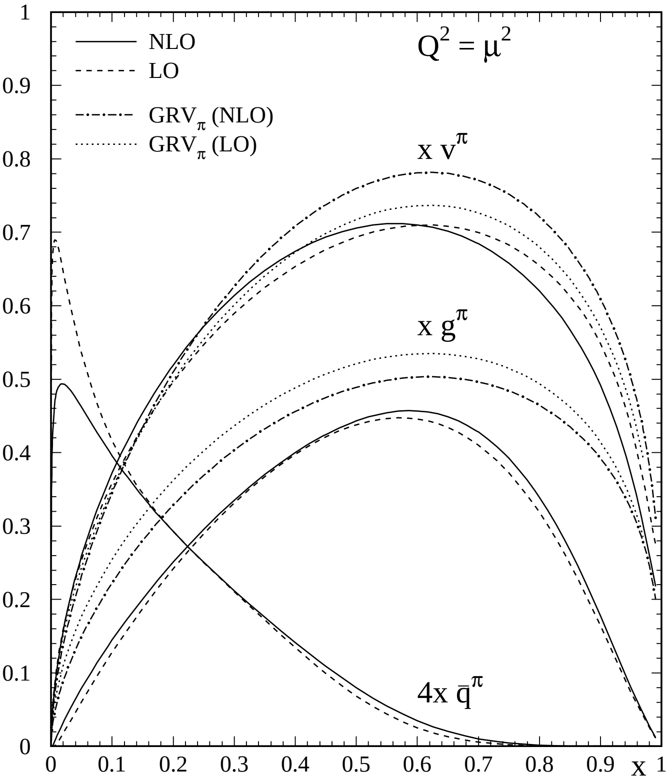

Fig. 1. The valence and valence–like input distributions $xf^\pi(x,Q^2=\mu^2)$ with $f = v$, $\bar{q}$, g as compared to those of GRV$_\pi$ [5]. Notice that GRV$_\pi$ employs a vanishing SU(3)$_{\rm flavor}$ symmetric $\bar{q}^{\,\pi}$ input at $\mu^2_{\rm LO} = 0.25$ GeV^2 and $\mu^2_{\rm NLO} = 0.3$ GeV^2 [5]. Our present SU(3)$_{\rm flavor}$ broken sea densities refer to a vanishing s^π input in (3), as for GRV$_\pi$ [5]

For completeness let us mention that our basic predictions (5) for the valence–like gluon and sea densities at $Q^2 = \mu^2$, as shown in Fig. 1, can be simply parametrized in Bjorken–x space : in LO at $Q^2 = \mu^2_{\rm LO} = 0.26$ GeV^2

$$x\,g^\pi(x,\mu^2_{\rm LO}) = 7.326\,x^{1.433}(1-1.919\,\sqrt{x}+1.524\,x) \times(1-x)^{1.326}$$
$$x\,\bar{q}^{\,\pi}(x,\mu^2_{\rm LO}) = 0.522\,x^{0.160}(1-3.243\,\sqrt{x}+5.206\,x) \times(1-x)^{5.20}\,, \tag{14}$$

whereas in NLO at $Q^2 = \mu^2_{\rm NLO} = 0.40$ GeV^2 we get

$$x\,g^\pi(x,\mu^2_{\rm NLO}) = 5.90\,x^{1.270}(1-2.074\,\sqrt{x}+1.824\,x) \times(1-x)^{1.290}$$

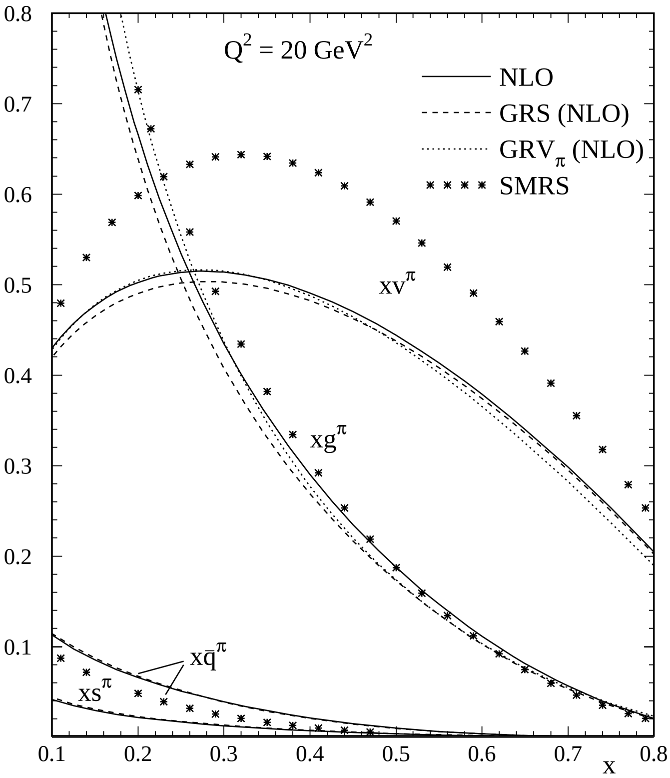

Fig. 2. Comparison of our NLO valence distribution at $Q^2 = 20\ \mathrm{GeV}^2$ with the one of GRV_π [5] and GRS [6]. This density plays the dominant role for describing presently available πN Drell–Yan dimuon production data. For illustration, the gluon and sea densities are shown as well. The $\mathrm{SU(3)}_{\mathrm{flavor}}$ symmetric GRV_π sea $\bar{q}^{\,\pi} = s^\pi$ is not shown, since it is similar to s^π of our present analysis and of GRS which are all generated from a vanishing input at $Q^2 = \mu^2$, cf. (3). The SMRS [3] results refer also to a $\mathrm{SU(3)}_{\mathrm{flavor}}$ symmetric sea $\bar{q}^{\,\pi} \equiv \bar{u}^{\,\pi^+} = d^{\pi^+} = s^\pi = \bar{s}^{\,\pi}$

$$x\,\bar{q}^{\,\pi}(x,\mu^2_{\mathrm{NLO}}) = 0.417\,x^{0.207}(1 - 2.466\,\sqrt{x} + 3.855\,x) \times (1-x)^{4.454}. \tag{15}$$

Finally, Fig. 3 shows our resulting predictions for $x\,g^\pi(x,Q^2)$ and $x\,\bar{q}^{\,\pi}(x,Q^2)$ as compared to the former GRV_π results [5]. The GRV_π results for $x\,\bar{q}^{\,\pi}$ are significantly steeper and softer for $x \gtrsim 0.01$ due to the vanishing $\mathrm{SU(3)}_{\mathrm{flavor}}$ symmetric (light) sea input $x\,\bar{q}^{\,\pi}(x,\mu^2) = 0$, in contrast to our present approach [6] based on a more realistic finite light sea input in (5). The valence–like gluon and sea inputs at $Q^2 = \mu^2$, which become (vanishingly) small at $x < 10^{-2}$, are also shown in Fig. 3. This illustrates again the purely dynamical origin of the small–x structure of gluon and sea quark densities at $Q^2 > \mu^2$. Our predictions for $s^\pi = \bar{s}^{\,\pi}$, as evolved from the vanishing input in (3), are not shown in the figure since they practically coincide with $\bar{q}^{\,\pi}(x,Q^2)$ of GRV_π shown in Fig. 3 which also results from a vanishing input [5]. Simple analytic parametrizations of our LO and NLO predictions for $f^\pi(x,Q^2)$ are given in the Appendix.

To conclude let us recall that an improvement of $f^\pi(x,Q^2)$ is particularly important in view of its central role in the construction of the photon structure function and the photonic parton distributions [14–18]. Furthermore, recent (large rapidity gap) measurements of leading proton and neutron production in deep inelastic scattering at HERA [19] allow, under certain (diffractive) model assumptions, to constrain and test the pion structure functions for the first time at far smaller values of x (down to about 10^{-3}) than those attained from fixed target πN experiments.

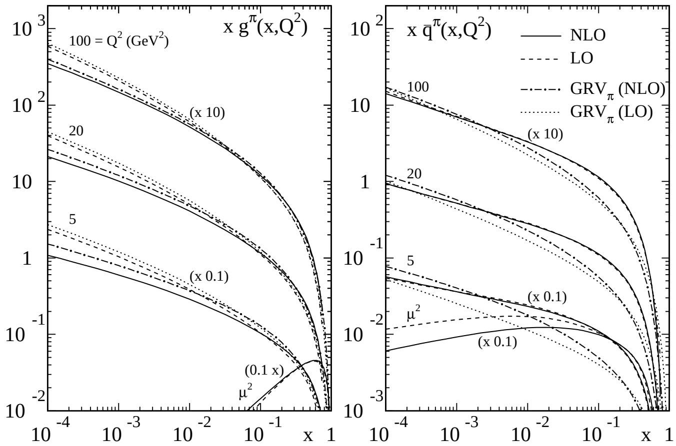

Fig. 3. The small–x predictions of our radiatively generated pionic gluon and sea–quark distributions in LO and NLO at various fixed values of Q^2 as compared to those of GRV_π [5]. The valence–like inputs, according to (5) as presented in Fig. 1, are shown for illustration by the lowest curves referring to μ^2. The predictions for the strange sea density $s^\pi = \bar{s}^{\,\pi}$ are similar to the GRV_π results for $\bar{q}^{\,\pi}$. The results are multiplied by the numbers indicated in brackets

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Appendix

A. Parametrization of LO parton distributions
Defining [9]

$$s \equiv ln\,\frac{ln\,[Q^2/(0.204\,\mathrm{GeV})^2]}{ln\,[\mu^2_{\mathrm{LO}}/(0.204\,\mathrm{GeV})^2]} \tag{A.1}$$

to be evaluated for $\mu^2_{\mathrm{LO}} = 0.26\ \mathrm{GeV}^2$, all our resulting pionic parton distributions can be expressed by the following simple parametrizations, valid for $0.5 \lesssim Q^2 \lesssim 10^5\ \mathrm{GeV}^2$ (i.e. $0.31 \leq s \lesssim 2.2$) and $10^{-5} \lesssim x < 1$. For the valence distribution we take

$$x\,v^\pi(x,Q^2) = N\,x^a(1 + A\sqrt{x} + Bx)(1-x)^D \tag{A.2}$$

with

$$N = 1.212 + 0.498\,s + 0.009\,s^2$$
$$a = 0.517 - 0.020\,s$$

$$A = -0.037 - 0.578\,s$$
$$B = 0.241 + 0.251\,s$$
$$D = 0.383 + 0.624\,s\,. \quad \text{(A.3)}$$

The gluon and light sea–quark distributions are parametrized as

$$x\,w^\pi(x,Q^2) = \left[x^a\left(A + B\sqrt{x} + Cx\right)\left(ln\,\frac{1}{x}\right)^b + s^\alpha\,\exp\left(-E + \sqrt{E' s^\beta ln\,\frac{1}{x}}\right)\right](1-x)^D. \quad \text{(A.4)}$$

For $w = g$

$$\begin{aligned} \alpha &= 0.504, & \beta &= 0.226, \\ a &= 2.251 - 1.339\,\sqrt{s}, & b &= 0, \\ A &= 2.668 - 1.265\,s + 0.156\,s^2, & B &= -1.839 + 0.386\,s, \\ C &= -1.014 + 0.920\,s - 0.101\,s^2, & D &= -0.077 + 1.466\,s, \\ E &= 1.245 + 1.833\,s, & E' &= 0.510 + 3.844\,s\,, \end{aligned} \quad \text{(A.5)}$$

and for the light sea $w = \bar{q}$

$$\begin{aligned} \alpha &= 1.147, & \beta &= 1.241, \\ a &= 0.309 - 0.134\,\sqrt{s}, & b &= 0.893 - 0.264\,\sqrt{s}, \\ A &= 0.219 - 0.054\,s, & B &= -0.593 + 0.240\,s, \\ C &= 1.100 - 0.452\,s, & D &= 3.526 + 0.491\,s, \\ E &= 4.521 + 1.583\,s, & E' &= 3.102\,. \end{aligned} \quad \text{(A.6)}$$

The strange sea distribution $s^\pi = \bar{s}^{\,\pi}$ is parametrized as

$$x\bar{s}^{\,\pi}(x,Q^2) = \frac{s^\alpha}{(ln\,\frac{1}{x})^a}\left(1 + A\sqrt{x} + Bx\right)(1-x)^D \times\exp\left(-E + \sqrt{E' s^\beta ln\frac{1}{x}}\right) \quad \text{(A.7)}$$

with

$$\begin{aligned} \alpha &= 0.823, & \beta &= 0.650, \\ a &= 1.036 - 0.709\,s, & A &= -1.245 + 0.713\,s, \\ B &= 5.580 - 1.281\,s, & D &= 2.746 - 0.191\,s, \\ E &= 5.101 + 1.294\,s, & E' &= 4.854 - 0.437\,s\,. \end{aligned} \quad \text{(A.8)}$$

B. Parametrization of NLO($\overline{\text{MS}}$) parton distributions
Defining [9]

$$s \equiv ln\,\frac{ln\,[Q^2/(0.299\,\text{GeV})^2]}{ln\,[\mu^2_{\text{NLO}}/(0.299\,\text{GeV})^2]} \quad \text{(A.9)}$$

to be evaluated for $\mu^2_{\text{NLO}} = 0.40\ \text{GeV}^2$, our NLO predictions can be parametrized as the LO ones and are similarly valid for $0.5 \lesssim Q^2 \lesssim 10^5\ \text{GeV}^2$ (i.e. $0.14 \lesssim \text{s} \lesssim 2.38$) and $10^{-5} \lesssim x < 1$. The valence distribution is given by (A.2) with

$$\begin{aligned} N &= 1.500 + 0.525\,s - 0.050\,s^2 \\ a &= 0.560 - 0.034\,s \\ A &= -0.357 - 0.458\,s \\ B &= 0.427 + 0.220\,s \\ D &= 0.475 + 0.550\,s\,. \end{aligned} \quad \text{(A.10)}$$

The gluon and light sea distributions are parametrized as in (A.4) where for $w = g$

$$\begin{aligned} \alpha &= 0.793, & \beta &= 1.722, \\ a &= 1.418 - 0.215\sqrt{s}, & b &= 0, \\ A &= 5.392 + 0.553\,s - 0.385\,s^2, & B &= -11.928 + 1.844\,s, \\ C &= 11.548 - 4.316\,s + 0.382\,s^2, & D &= 1.347 + 1.135\,s, \\ E &= 0.104 + 1.980\,s, & E' &= 2.375 - 0.188\,s\,. \end{aligned} \quad \text{(A.11)}$$

and for the light sea $w = \bar{q}$

$$\begin{aligned} \alpha &= 1.118, & \beta &= 0.457, \\ a &= 0.111 - 0.326\,\sqrt{s}, & b &= -0.978 - 0.488\,\sqrt{s}, \\ A &= 1.035 - 0.295\,s, & B &= -3.008 + 1.165\,s, \\ C &= 4.111 - 1.575\,s, & D &= 6.192 + 0.705\,s, \\ E &= 5.035 + 0.997\,s, & E' &= 1.486 + 1.288\,s\,. \end{aligned} \quad \text{(A.12)}$$

The strange sea distribution is parametrized as in (A.7) with

$$\begin{aligned} \alpha &= 0.908, & \beta &= 0.812, \\ a &= -0.567 - 0.466\,s, & A &= -2.348 + 1.433\,s, \\ B &= 4.403, & D &= 2.061, \\ E &= 3.796 + 1.618\,s, & E' &= 0.309 + 0.355\,s\,. \end{aligned} \quad \text{(A.13)}$$

Let us recall that in the light quark sector $u_v^{\pi^+} = \bar{d}_v^{\,\pi^+} = \bar{u}_v^{\,\pi^-} = d_v^{\pi^-}$, $\bar{u}^{\,\pi^+} = d^{\pi^+} = u^{\pi^-} = \bar{d}^{\,\pi^-}$ and $f^{\pi^0} = (f^{\pi^+} + f^{\pi^-})/2$.

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